# Errata for Stakgold Boundary Value Problems of Mathematical Physics <br> Phil Lucht <br> Rimrock Digital Technology, Salt Lake City, Utah 84102 <br> last update: January 3, 2012 <br> rimrock@xmission.com 

These errata are for the following two books, both blue-covered hardbacks in the Macmillan Series in Advanced Mathematics and Theoretical Physics,
I. Stakgold, Boundary Value Problems of Mathematical Physics

Volume I, 1967, sixth printing 1972 (Macmillan, London) (Chapters 1 through 4)
Volume II, 1968, second printing 1971 (Macmillan, London) (Chapters 5 through 8)
The books were reprinted in 2000 in a SIAM edition with corrections, but some errata managed to escape detection as noted below.

These "errata" are just items I happened to notice while working my way through the first 7 chapters of this two volume set (Chapter 8 will have to wait). I bought these books in 1972 while taking Math 224 at UC Berkeley, but only recently was I able to find time to read them!

My reasons for creating this errata list are as follows:
(1) these wonderful books should be preserved indefinitely in the best possible working condition.
(2) The books are excellent but are also somewhat difficult in places, requiring the reader to do many calculational steps between "waypoints" given by the author. When there are errors in those waypoints, the student having a finite amount of time may give up or at least be quite discouraged.
(3) Someone who borrows these volumes from a library is likely to get the original editions, not the new editions which have many items corrected.
(4) Even the new 2000 editions continue to have some errata.
(5) I could find no existing errata on line.

I understand probably better than most why the author decided to put out a more condensed version of the book ( called Green's Functions and Boundary Value Problems now in a 3rd edition Wiley 2011) which "could be covered in a one-year course" as he says in his Preface. But if you really want to know the nuts and bolts inside the machine I recommend the two-volume set because it gives many examples of each presented technique and these examples do the ultimate teaching.

Any of my errata of course could be erroneous, for which I apologize ahead of time. Some errata are trivial typos hardly worth mentioning, but since I marked them in my book, they might as well get recorded here.

I had a friendly email exchange with Prof. Stakgold who is retired but still working (and surely playing lots of bridge) in his mid 80's at U.C. San Diego, see http://ccom.ucsd.edu/people/faculty.php. He said it was unlikely that there would be any future editions of these two books, but would include these errata if there were.

An asterisk * after the page number indicates an erratum not fixed in the 2000 SIAM edition.

## Volume I.

## Chapter 1

none

## Chapter 2

p 136* 9th line down should say We are to show $T\left[x_{n}\right] \rightarrow T[x] / /$ not $\rightarrow[T x]$
p 146 4th line from the bottom should say in (2.34) below // not (2.33)
p 163 5th line from bottom should say $\quad \mathrm{I}_{4}\left[\varphi_{1}\right]=\left|\mu_{1}\right| \quad / /$ not $=\left|\lambda_{1}\right|$
p 176 first line should say $\quad \int_{\mathbf{0}}^{\mathbf{t}} \mathrm{du}\left[\mathrm{y}(\mathrm{u}) \ldots \quad / /\right.$ not $\int_{\mathbf{0}}^{\mathbf{t}} \mathrm{dt}[\mathrm{y}(\mathrm{u}) \ldots$
p 176* equ (IV) at bottom should say $\quad y(1)=0 \quad / / \operatorname{not} x(1)=0$
p 184 mid page should say The second boundary...if $0=\int_{\mathbf{0}}^{\mathbf{1}} \mathrm{e}^{-\lambda \xi} \mathrm{f}(\xi) \mathrm{d} \xi$. $/ /$ not $\int_{\mathbf{0}}^{\mathbf{t}}$

## Chapter 3

p 204 2nd last line should say ..for the solution of the inhomogeneous ... // not homogeneous

## Chapter 4

p 299* equ (4.83a) $\beta_{0}$ in last term should be $b_{0}$ to match $b_{0}$ in (4.82a) and earlier discussion. This was changed in 2000 so that all terms show $\beta_{0}$, but all should show $b_{0}$.
p 303* last equ should have $\quad \hat{g}=-\left(\alpha_{1}{ }^{2}+\alpha_{2}{ }^{2}\right)^{-1}\left[\psi \ldots . \quad / /\right.$ not $\left(\alpha_{1}{ }^{2}+\alpha_{2}{ }^{2}\right)[\psi$
p 314* 2nd para should begin Each possible... of A in (4.125a) yields // not of A is (4.125a)
p 315 1st equ bracket factor should be $[\gamma+\log (\sqrt{\lambda} / 2)] \quad / / \operatorname{not}[(\gamma+\log (\sqrt{\lambda})) / 2]$
p 315* 1st equ in Ex 4.26 should have $\rho$ replaced by $x$ in two places so then has $x_{<}$and $x_{>}$.

## Volume II

## Chapter 5

$\left.\begin{array}{lll}\text { p } 38^{*} & \text { 2nd equ in Ex } 5.18 \text { should have factor } & (-\mathrm{iu})^{\mathrm{p}}\end{array}\right]$ not $\left.(-\mathrm{iu})^{\mathrm{n}}\right)$

## Chapter 6

p 94 below equation number (6.10) should read expression // not exprersion
p 131 equ (6.75) should begin with $g(x \mid \xi)=\quad / / \operatorname{not} g(x) \mid \xi)=$
p 132 Theorem 4 in bold type should read Theorem 3 (Theorem 4 is on page 133).
p 144 Ex 6.30, 6th line up from bottom should say $Z=\sqrt{z} R \quad / / \operatorname{not} Z=z R$
p 145* 2nd equation should say $u_{m, n, k}=(1 / \sqrt{r}) J_{n+(1 / 2)} \quad / / \operatorname{not}(1 / r) J_{n+(1 / 2)}$
p 147 1st equ after EXAMPLE should end with $=-(1 / 4 \pi) \log \left(1+\mathrm{r}^{2}-2 \mathrm{r} \cos \theta\right) / / \log$ is missing
p 158 2nd equ on page (one after Thus) is missing a factor $\sin (m \pi x / a)$, but OK in (6.117).
p $161^{*} \mathrm{Eq}(6.122)$ integration variable should be d $\alpha$ and not dx.
p 165* 2nd last equ begins $w \bar{w}=E \bar{E}$ ( num/denom) : last signs in both num and denom should be + instead of - .

p 179* 3rd line of text should say According to (6.195) in Exercise 6.47
and not According to (6.192) in Exercise 6.48 two typos here the second typo is fixed in the 2000 SIAM edition
p 179* 5th equ, last bracket should be $\left[c_{0}-(1 / 2 \pi) \log a\right] \quad / / \operatorname{not}\left[c_{0}(1 / 2 \pi) \log a\right]$
$\mathrm{p} 183 *$ 4th equ bracket should be $\left[\frac{\mathrm{R}^{2}-\mathrm{r}_{0}{ }^{2}}{2 \mathrm{a}}+\frac{\mathrm{a}}{2}\right] \quad / / \operatorname{not}\left[\frac{\mathrm{R}^{2}-\mathrm{r}_{0}{ }^{2}}{2}+\frac{\mathrm{a}}{2}\right]$
p $183 * 5$ th equ should say $\mathrm{I}(\mathrm{s})=-\frac{1}{4 \pi \mathrm{aR}^{3}}[..] \quad \quad / / \operatorname{not} \mathrm{I}(\mathrm{s})=-\frac{1}{4 \pi \mathrm{R}^{3}}[.$.
p 187* last equ on page should read $\left.u(s-)-u_{i}(s-)=\quad / / \operatorname{not} u(s-)-u_{i} s-\right)=$

## Chapter 7

p 213 1st sentence after heading should say for the heat equation // not equaton p 218 6th line of text should say Alternatively, we use (7.57) // not (7.47)
p 218 2nd last equ should have $\widetilde{\mathrm{q}}(\mathrm{x}, \mathrm{s})+\mathrm{f}(\mathrm{x}) \quad / /$ not $\widetilde{\mathrm{g}}(\mathrm{x}, \mathrm{s})$
p 230 1st equ should have factor (2/l) before the $\Sigma \quad / /$ as in equ (7.50)
p 237* 4th line of text should say of strength $4 \pi / 3$ not strength $4 \pi$
p 240 Ex 7.10 equ should have factor $\frac{4 \pi n}{\ell^{2}} \quad / / \operatorname{not} \frac{4 \pi n}{\ell}$
p 242* 3rd line of text above Ex 7.17 should say Fourier sine transform // not cosine transform
p 242* last equ: The two functions $\mathrm{H}_{(0)}$ and $\mathrm{Z}_{0}$ are strangely in reverse order. Everywhere else in the book the $x_{<}$function is put on the left of the $x_{>}$function.

| p 243 the 4 th equ should have $-\delta(x-\xi) / x$ on the left | $/ / \operatorname{not} \delta(x-\xi) / x$ |
| :--- | :--- |
| p 243 | the 6th equ left side should be $-f(x)$ |

p 243 the 6th equ should have $F_{w}(\mu)$ added to the integrand
p 248* 4th equ should have factors $f_{2, k}$ and then $f_{1, k} \quad / / \operatorname{not} f_{1, k}$ then $f_{2, k}$
p 250 last integral in (7.122) should have $a+(1 / 2) \int / / \operatorname{not}-(1 / 2) \int$
p 252* 4th equ, the integration should be $\int_{\mathbf{0}}^{\mathbf{t 0}} \quad / / \operatorname{not} \int_{\mathbf{0}}^{\mathbf{t}}$
p 254 the $K=$ equation should have $f_{1}(x)$ in the integrand // not $f(x)$
p 258 the $\mathrm{w}_{\mathrm{k}}=$ triplet above (7.137) should have $\sinh$ and sin // not cosh and cos
p 260 1st of equ (7.141) triplet, should have $-\nabla^{2} v=0 \quad / /$ not $-\partial^{2} v / \partial x^{2}=0$
p 270* 2nd line should read in (7.163) and // not in (7.164) and p 271* equ (7.170) is missing the $H_{\alpha}{ }^{(1)}\left(\sqrt{\lambda r_{>}}\right)$factor which appears in (7.169)
p 273* equ (7.176) should contain factor $\tilde{f}(\gamma) \quad / / \operatorname{not} \bar{f}(\gamma)$
p 273 text line above equ 7.177 should say $\quad$ by $K_{i \gamma}(\mathrm{kr}) \quad / /$ not by $\mathrm{rK}_{\mathrm{i}}(\mathrm{kr})$
p 273* last equation should have $\sinh (\pi \gamma) \quad / / \operatorname{not} \sin (\mathrm{k} \gamma \pi)$
p 275* Equ after At $\varphi=0$ we have should have $\sinh (\gamma \pi) \quad / / \operatorname{not} \sin (\gamma \pi)$
p 275* Equ after At $\varphi=2 \pi$, should have $\sinh (\gamma \pi) \quad / / \operatorname{not} \sin (\gamma \pi)$
p 275* the equ after We deduce that should read

$$
\tilde{v}=-\frac{\cosh [\gamma(\varphi-\pi)]}{\gamma \sinh (2 \gamma \pi)} \mathrm{K}_{\mathrm{i} \gamma}\left(\mathrm{kr}_{0}\right) \quad \text { // not the very different expression shown. }
$$

Here are a few more waypoints leading to (7.183):

$$
\begin{aligned}
\mathrm{v}\left(\mathrm{r}, \varphi, \mathrm{r}_{0}\right) & =-\left(1 / \pi^{2}\right) \int_{0}^{\infty} \mathrm{d} \gamma \mathrm{~K}_{\mathrm{iY}}(\mathrm{kr}) \mathrm{K}_{\mathrm{i} \mathrm{\gamma}}\left(\mathrm{kr}_{0}\right) \operatorname{ch}[\gamma(\varphi-\pi)] / \operatorname{ch}(\gamma \pi) \quad \varphi_{0}=\pi \\
\mathrm{g}\left(\mathrm{r}, \varphi \mid \mathrm{r}_{0}, \pi\right) & =\left(1 / \pi^{2}\right) \int_{0}^{\infty} \mathrm{d} \gamma \mathrm{~K}_{\mathrm{i}}(\mathrm{kr}) \mathrm{K}_{\mathrm{i}}\left(\mathrm{kr}_{0}\right)\{\operatorname{ch}[\gamma(|\varphi-\pi|-\pi)]-\operatorname{ch}[\gamma(\varphi-\pi)] / \operatorname{ch}(\pi \gamma)\} \\
& =-(\mathrm{i} / 2 \pi) \int_{-\infty}^{\infty} \mathrm{d} \gamma \mathrm{I}_{-\mathrm{i}}\left(\mathrm{kr}_{<}\right) \mathrm{K}_{-\mathrm{i}}\left(\mathrm{kr}_{>}\right)\{\operatorname{th}(\gamma \pi) \operatorname{ch}(\gamma(|\varphi-\pi|)-\operatorname{sh}(\gamma(|\varphi-\pi|)\}
\end{aligned}
$$

p 277 equ (7.192) the radical in the left side exponent should be $\sqrt{\mathrm{r}^{2}+\mathrm{z}^{2}}$, not $\sqrt{\mathrm{r}^{2}-\mathrm{z}^{2}}$
p 279* equ after (7.200), the radical exponent should be $\sqrt{\mathrm{r}^{2}+\mathrm{z}^{2}}$, not $\sqrt{\mathrm{r}^{2}-\mathrm{z}^{2}}$. This is OK in the following equation.
p 289* first equ in the In Region I: should read $\quad 0,0<t<\mathrm{R} \quad / /$ the 0 , is missing In other words it should look the way it looks for Region II.
p 292 last equation on page, the $r$ in the first parentheses is upside down.
p $312 *$ first line after EXAMPLE: should read $\quad$ let $a(\omega)=1 /\left(1+\omega^{2}\right) \quad / / \operatorname{not} 1 / 1+\omega^{2}$
p 317* start of 2nd paragraph should read let $\mathrm{a}(\omega)=1 /\left(1+\omega^{2}\right) \quad / /$ not $1 / 1+\omega^{2}$ (perhaps the above two items represented a valid notation in use at the time, but seems unclear) p 318 4th equ, left side should be $b_{+}(\omega) / b_{-}(\omega) \quad / /$ not $b_{+}(\omega)+b_{-}(\omega)$ p 319 first equ should have 2 in the numerator // not 1 in numerator p 321* 2nd line of text should say the integral by closing the contour // not by choosing the contour p 322 equ (7.300), the factor $1 /(2 \xi)$ in the first integral should be $1 /(2 \pi)$ p 325* first line of text should say Setting $\alpha=-i R \quad / /$ not $\alpha=i R$ p 328* second equation on page: factor of $k$ should not be present (in fact, $k=0$ ) p 330 Ex 7.54, 2nd equ should have leading factor $\pi / \sqrt{\mathrm{p}+\mathrm{k}} \quad / /$ not $\pi /(\mathrm{p}+\mathrm{k})$ p 330* In Ex 7.56 should have " plane wave $e^{i k y} " \quad / /$ not " plane wave e ${ }^{\text {ky }}$ " p 330* Last equation on page should read: (factor of 2 in first term; presence of second term 2 k )

$$
\mathrm{I}(\mathrm{x})=(2 \sqrt{\mathrm{k}} / \pi) \int_{\mathbf{k}}^{\infty} \mathrm{e}^{-\mathrm{xR}}(1 / \mathrm{R}) \sqrt{\mathrm{R}-\mathrm{k}} \mathrm{dR}+2 \mathrm{k}
$$

Evaluation then gives $\mathrm{I}(\mathrm{x})=2 \mathrm{k} \operatorname{erf}(\sqrt{\mathrm{kx}})+2 \sqrt{\mathrm{k} / \pi} \mathrm{e}^{-\mathrm{kx}} / \sqrt{\mathrm{x}}$
from which the 2nd last equation on page 330 is verified by Laplace transform with $\mathrm{s}=-\mathrm{i} \omega$.
p 393 Last term in first equation should be $+\left[1 /\left(r^{2} \sin ^{2} \theta\right)\right] \partial_{\varphi}{ }^{2} u \quad / / \operatorname{not}\left[1 /\left(r^{2} \sin \theta\right)\right] \partial_{\varphi}{ }^{2} u$
p 397* First equation should end $=\left(1 / N_{m, n}\right) \delta\left(r-r_{0}\right) \overline{\mathrm{Y}}^{m}{ }_{n}\left(\theta_{0}, \varphi_{0}\right) \quad / / \operatorname{not}\left(1 / N_{m, n}\right) \bar{Y}^{m}{ }_{n}\left(\theta_{0}, \varphi_{0}\right)$
p 400 Equation (B.5) should not have n ! in the denominator

